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Surrogate Modeling-Based Optimization of SAGD Processes

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Abstract

This paper presents a solution methodology for the optimization of geometrical and operational parameters of SAGD processes in a heterogeneous and multiphase petroleum reservoir. The optimization refers to the maximization or minimization of performance measures such as net present value, cumulative oil production or cumulative steam injected.

The solution methodology includes the construction of a "fast surrogate" of an objective function whose evaluation involves the execution of a time-consuming mathematical model (i.e. reservoir numerical simulator) based on neural networks, DACE modeling, and adaptive sampling. Using adaptive sampling, promising areas are searched considering the information provided by the surrogate model and the expected value of the errors.

The proposed methodology provides a global optimization method, hence avoiding the potential problem of convergence to a local minimum in the objective function exhibited by the commonly Gauss-Newton methods. Furthermore, it exhibits an affordable computational cost, is amenable to parallel processing, and is expected to outperform other general-purpose global optimization methods such as, simulated annealing, genetic algorithms, and pattern search methods.

The methodology is evaluated using a case study with vertical spacing, steam injected enthalpy, injection pressure and subcooling as the sought parameter values in a SAGD process that optimize a weighted sum of cumulative oil production and cumulative steam injected for a selected reservoir. From the results, it is concluded that the methodology can be used effectively and efficiently for the optimization of SAGD processes. In addition, the optimization approach holds promise to be useful in the optimization of objective functions involving the execution of computationally expensive reservoir numerical simulators, such as those found, not only in oil recovery processes, but also in other areas of petroleum engineering (e.g. hydraulic fracturing).

Introduction

There is considerable interest in effective oil recovery mechanisms for heavy oil and bitumen due to the decline of conventional oil reserves, and the estimated magnitude of these resources worldwide (approximately 6 trillion bbl). A major part of these resources are located in Venezuela, Canada and the United States.[1]

While the use of horizontal wells has improved the recovery of heavy oil, the ultimate oil recovery remains unsatisfactory due to the low mobility of the crude at reservoir conditions. Different alternatives have been proposed in the last three decades for improving the flowing capacity of heavy oil and improve oil recovery. Examples of these alternatives are, cyclic steam stimulation (CSS), steam drive, in situ combustion, and SAGD. The latter could be effective even in reservoirs containing highly viscous oil or bitumen[2] and have proven to be economically viable at a variety of pilot[3,4] and commercial recovery projects,[3] typically achieving oil recoveries of over 50% from the well pattern with a steam/oil ratio of 2.5 to 4.[2] See the work by Butler[3] and the references contained in it for details of the SAGD concept and mechanisms.

The performance of the SAGD process can be significantly affected by the selection of the geometrical and operational parameters. Examples of the former are the vertical spacing, lengths of the producer and injector wells, and the horizontal separation between well pairs; the latter include parameters such as steam injected enthalpy, injection pressure and subcooling. Even though there have been significant contributions regarding screening of reservoir candidates,[1,5] theoretical aspects,[3,6] analytical and numerical modeling,[7,8,9] laboratory experiments,[10,11] the optimal or near optimal selection of the aforementioned parameters have been addressed only by a few sensitivity studies.[12,13]

Kamath et al.[12] using a numerical two-dimensional model that accounts for reservoir heterogeneities conducted a sensitivity study of a SAGD process which considers the relative influence with respect to a base case of different parameters such as porosity, absolute permeability, steam temperature, steam quality, horizontal well length, injector/producer spacing, shale barriers, and lateral well spacing, among others. The study establishes percent recovery, and oil/steam ratio, as performance measures. Kisman and Yeung[13] performed a similar study using a two dimensional base case numerical model that quantifies the relative influence of factors such as thermal conductivity, flow barriers, oil viscosity, relative permeability, solution gas, well placement, among others. Note that these are both sensitivity studies that do not address the formal optimal setting of geometrical and operational parameters.

This paper presents a solution methodology called NEGO (neural network based efficient global optimization) for the optimization of the geometrical and operational parameters in a SAGD process, such that a given performance measure is minimized. The solution methodology includes the construction of a "fast surrogate" of an objective function whose evaluation involves the execution of a time-consuming mathematical model (i.e. reservoir numerical simulator) based on neural networks, DACE[14] modeling, and adaptive sampling. Using adaptive sampling, promising areas are searched considering the information provided by the surrogate model and the expected value of the errors.

The DACE surrogate model is initially constructed using sample data generated from the execution of mathematical models with parameters given by a latin hypercube experimental (LHC) design, and a neural network, and provides error estimates at any point. Additional points are obtained balancing the exploitation of the information provided by the surrogate model (where the surface is minimized) with the need to improve the surface (where error estimates are high). The proposed methodology provides a global optimization method, hence avoiding the potential problem of convergence to a local minimum in the objective function exhibited by the commonly used Gauss-Newton methods,[15,16] and computational cost involved in numerically estimating derivatives, and in the step by step movement along given trajectories. Furthermore, it exhibits an affordable computational cost, is amenable to parallel processing, and is expected to outperform other general purpose global optimization methods such as, simulated annealing, genetic algorithms[17,18] and pattern search methods.[19]

Problem Definition

The optimization of SAGD processes is a complex task. The complexity is associated with a time consuming and limited number of objective function (performance measure) evaluations, a potentially high number of parameters, and a non-linear solution space. Performance measures such as net present value, cumulative oil production, and cumulative steam injection, require computationally expensive reservoir numerical simulations restricted in number given the time constraints typically present in the oil industry. The number of geometrical (e.g. vertical and horizontal spacing, and wells length) and operational parameters (e.g. subcooling, steam injected enthalpy, injection temperature, etc.) to be considered may be significant, and solutions may be needed under different economic and reservoir/oil property scenarios. In addition, the non-linear nature of the process makes not possible the identification of optimal settings through sensitivity studies (as it is usually performed). Formally, it can be written as:

$$\begin{aligned} &\textit{find } x \in X \subseteq R^p \\ &\text{such that} \\ &f(x) \text{ is minimized} \end{aligned}$$

where f is a mathematical function (objective function) of x, the geometrical and operational parameter vector, and X is the set constraint. Hence, the problem of interest is one of finding the vector of parameters that minimized a given performance measure of a SAGD process subject to a set constraint.

Solution Methodology

The proposed solution approach called NEGO,[20] neural-network based efficient global optimization, is an improved version of the EGO algorithm[21] for the optimization of computationally expensive black-box functions.

The proposed solution methodology involves the following four steps:

1. Construct a sample of the parameter space using the latin hypercube method. The latin hypercube sampling procedure has been shown to be very effective for selecting input variables for the analysis of the output of a computer code.[22]
2. Conduct mathematical simulations using the sample from the previous step and obtain the objective function values.
3. Construct a parsimonious neural network (multilayer perceptron) using the data from the previous step. The purpose of this neural network is to capture the general trends observed in the data; no rigorous performance criterion is placed on the neural network.
4. Construct a DACE model for the residuals, that is, the difference between the observed objective function values, and the neural network responses using the sample data. These models provide not only estimates of the residuals value but also of the respective errors. The surrogate model for the evaluation of the objective function is the sum of the neural network and DACE models. Details of this step will be given later in this section.
5. Additional points are obtained balancing the exploitation of the information provided by the surrogate model (where the surface is minimized) with the need to improve the surface (where error estimates are high), until a stopping criterion has been met. This balance is achieved by sampling where a figure of merit is maximized. Details of the figure of merit will be given later in this section.

DACE models. These models owe their name, design and analysis of computer experiments, to the title of an article that popularized the approach.[14] These models suggest to estimate deterministic functions as shown in Eq. 2.

$$y(x_j) = \mu + \varepsilon(x_j) \quad (2)$$

where, f is the function to be modeled, μ is the mean of the population, and ε is the error with zero expected value, and with a correlation structure given by Eq. 3.

$$\mathrm{cov}(\varepsilon(x_i), \varepsilon(x_j)) = \sigma^2 \exp\left(-\sum_{h=1}^{p} \theta_h \left(x_i^h - x_j^h\right)^2\right) \quad (3)$$

where, p denotes the number of dimensions in the vector x, σ, identifies the standard deviation of the population, and, θ_h is a correlation parameter, which is a measure of the degree of correlation among the data along the h direction.

Specifically, given a set of n input/output pairs (x, y), the parameters, μ, σ, and θ are estimated such that the likelihood function is maximized.[14] Having estimated these values, the function estimate for new points is given by Eq. 4.

$$\bar{y}(x) = \bar{\mu} + r' R^{-1}(y - 1\bar{\mu}) \quad (4)$$

where, the line above the letters denote *estimates*, r' identifies the correlation vector between the new point and the points used to construct the model, R is the correlation matrix among the n sample points, and L denotes an n-vector of ones.

The mean square error of the estimate is given by Eq. 5.

$$s^2\left(x^*\right) = \sigma^2\left[1 - r' R^{-1} r + \frac{\left(1 - L' R^{-1} r\right)}{L' R^{-1} L}\right] \quad (5)$$

The model is validated through a cross validation procedure, that essentially makes sure that the estimates using all but the point being tested and the actual response values are within an specified number of standard deviations. The original EGO algorithm may not cross-validate properly if there are trends in the data, in contrast to NEGO, which is expected to subtract any significant trends in the data.

The benefits of modeling deterministic functions using this probabilistic approach are: i) represents a best linear unbiased estimator, ii) interpolates the data, and iii) provides error estimates.

Figure of merit. With reference to **Fig 1**, there are two zones where it is desirable to add additional points. The zone (left) where the objective function is minimized and the zone (right) where there is a significant error in the prediction. Hence the figure of merit for adding sample points should be high in either of these situations. Specifically, the figure of merit[21] used in this work, is given by Eq. 6.

$$fom(x) = \left(f_{min} - \hat{f}\right)\Phi\left(\frac{f_{min} - \hat{f}}{s}\right) + s\phi\left(\frac{f_{min} - \hat{f}}{s}\right) \quad (6)$$

where, Φ and ϕ are the cumulative and density normal distribution functions, respectively; and $f_{\min}$ denotes the minimum current objective function value. Eq. 6 establishes the desired balance of sampling where the response surface (the predictor) is minimized (left term) and in zones where error estimates are high (right term). Note that the figure of merit makes reference to the objective function so it includes the sum of the output of both the neural network and the residual models.

This surface response approach for global optimization is expected to outperform competing methods, in terms of necessary computationally expensive objective function evaluations, to meet a stopping criterion. It can identify promising areas without the need of moving step by step along a given trajectory. In addition, by providing estimates of the errors at unsampled points, it is possible to establish a reasonable stopping criterion. Furthermore, provides a fast surrogate model that could be used to visualize the relationship between the sought parameters and the objective function values and to identify the relative significance of each of the parameters.

Implementation. The following case study was solved using an implementation of the NEGO algorithm developed by the authors[20] in Matlab.[24] The subproblems of finding near optimal values for maximizing likelihood and the figure of merit were solved using the DIRECT method[23]. Note that the solution of these subproblems do not require additional computationally expensive objective function evaluations. The reservoir numerical simulations were conducted using a commercial reservoir numerical simulator (EXOTHERM).[25]

Case Study

The NEGO algorithm was evaluated using a synthetic problem having geometrical and operational parameters: vertical spacing, injection pressure, steam-injected enthalpy and subcooling, with ranges as specified in **Table 1**. The objective function (given by Eq. 7) to be minimized is a weighted sum of normalized values of cumulative oil production (COP) and cumulative steam injected (CSI).

$$f(\bar{x}) = -\frac{3}{4}COP + \frac{1}{4}CSI \quad (7)$$

The weights (-0.75 for COP and +0.25 for CSI) reflect a preference structure and the intend to maximize COP and minimize CSI. The values of COP and CSI are calculated after a five (5) year production period.

An illustration of the 2D reservoir simulation grid under consideration and the coordinate system is depicted in **Fig. 2**. The grid is composed of 40x1x54 blocks in the x, y and z directions, respectively, with symmetry with respect to the z axis. The producer well is placed in the block denoted as (1,1,49), while the injector well is placed in a block within the blocks (1,1,35) to (1,1,47); both wells are of 1500 m length. The reservoir is at a depth of 500 m, has an initial pressure of 500 kPa, and initial oil and water saturation of 0.85 and 0.15, respectively. Furthermore, the porosity is assumed to be constant throughout the reservoir and equal to 0.2, the horizontal permeability is isotropic and equal to 1500 md and vertical permeability is equal to 450 md. The initial temperature of the reservoir is 15 ºC. Further details of the reservoir and fluid data are presented in **Tables 2 to 5 and Fig. 3.**

The neural network and DACE models were constructed using a sample of forty (40) points selected using a latin hypercube sampling procedure. Fifteen (15) additional points were added in the search of the optimum parameters.

Results and discussion

With reference to the case study, the parsimonious neural network has a 4x1x1 architecture with a mean square normalized error of 3.657E-02; all the points in the DACE model cross-validated within three times of the standard deviation.

The initial sample also shows (**Table 6**) the sensitivity of the objective function to the parameter selection with COP and CSI in the intervals [2.2E3 m^3, 154.8E3 m^3], and [5.0E3 m^3, 710.6E3 m^3], respectively. The minimum objective function value found within the initial sample (40 points) was –0.5418 which corresponds to a COP of 154.8E3 m^3 and a CSI of 592.5E3 m^3; the associated parameters values for vertical spacing, injection pressure, steam-injected enthalpy and subcooling are 11 m, 3756.6 kPa, 2578.4 kJ/kg and 29.7 ºC respectively.

Additional points (15) maximizing the figure of merit were added (see **Table 7**). From those points the best solution found (2^{nd} additional sampled point) observed an objective function value of -0.5537, that is slightly better than the corresponding to the initial sample (2.19% lower) with a COP of 156.9E3 m^3 and a CSI 589.0 E3 m^3 with similar parameter values. Changing parameter values with respect to the overall best solution found or extending the optimization process of the figure of merit did not improve the objective function value; all of which suggest the cited solution is in fact near optimal.

The parameters associated with the optimal or near optimal solution found could not have been anticipated because of the complex non–linear interaction among the selected parameters and the objective function. Selecting maximum parameter values results in 70% lower COP and 72% lower CSI; maximum parameter values for injection pressure, steam-injected enthalpy and subcooling, and minimum vertical spacing translates in 52% lower COP and 36% lower CSI; maximum parameter values for injection pressure, steam-injected enthalpy and subcooling, and the frequently used vertical spacing of 5 m resulted in 22% lower COP and 16% lower CSI; finally, mean parameter values provided 30% lower COP and 20% lower CSI. All of these alternatives provide higher objective function values.

Conclusions

- A global optimization method for the evaluation of the operational parameters of SAGD process called NEGO has been proposed. The method includes the construction of a “fast surrogate” of an objective function whose evaluation involves the execution of a time-consuming mathematical model (i.e. reservoir numerical simulator) based on neural networks, DACE modeling, and adaptive sampling. Using adaptive sampling, promising areas are searched considering the information provided by the surrogate model and the expected value of the errors.
- The results suggest that the NEGO algorithm can be used effectively and efficiently for improved oil recovery purposes. In addition, the optimization approach holds promise to be useful in the optimization of objective functions involving the execution of computationally expensive mathematical models (e.g. reservoir numerical simulators), such as those found, not only in oil recovery processes, but also in other areas of petroleum engineering (e.g. hydraulic fracturing).
- The NEGO algorithm is expected to outperform competing methods, in terms of computationally expensive objective function evaluations necessary to meet a stopping criterion. This is because it can identify promising areas without the need of moving step by step along a given trajectory. Furthermore, provides a fast surrogate model that could be used to visualize the relationship between the sought parameters and the objective function values and to identify the relative significance of each of the parameters.

Nomenclature

DACE = Design and analysis of computer experiment
x = Parameters vector
X = Set constraint
f = Objective function
$\hat{f}$ = NEGO objective function predictor
w_i = Weighting coefficients
μ = Mean of the population
ε = Error in the DACE model
p = Number of dimensions in the vector x
σ = Standard deviation of the population
θ_h = Correlation parameter
r = Correlation vector between the new point and the points used to construct the model
R = Correlation matrix between the n sample points
L = n-vector of ones
fom = Figure of merit
Φ = Cumulative normal distribution function
ϕ = Density normal distribution function
y = Residual function
$\bar{y}$ = DACE residual predictor
fmin = Current best function value
$s^2(x^*)$ = Mean square error of the predictor
COP = Cumulative oil production (m^3)
CSI = Cumulative steam injected (m^3)
COV = Covariance
BIS = Best initial solution
LHC = Latin hypercube

Subscript
h = Coordinate directions

Superscript
$*$ = New point
' = Transpose

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TABLE 1 – PARAMETER RESTRICTIONS (CASE STUDY)

Parameter	Description	Range Min	Range Max	Units
x_1	Vertical spacing of wells	3	15	m
x_2	Injection pressure	1000	4000	kPa
x_3	Steam – injected enthalpy	1980	2580	kJ/kg
x_4	Subcooling	5	30	°C

TABLE 2 - RESERVOIR DATA AND PETROPHYSICAL PROPERTIES (CASE STUDY)

	X	Y	Z	Units
Gridblocks	40	1	54	
Gridblock size	1.5	1500	1	m
Reservoir size	60	1500	54	m
Rock compressibility			1.00E-07	kPa^{-1}
Rock heat capacity			2390.00	kJ/m3/K
Rock thermal conductivity			147	kJ/D-m-K
Gas - Oil contact depth			160	m
Reservoir initial temperature			15	°C
Reservoir initial pressure			500	kPa
Oil:				
Thermal Capacity			1.88	kJ/kg/K
Thermal Expansion			7.0E-4	$ºC^{-1}$
Density			1029	Kg/m^3
Compresibility			7.2E-7	kPa^{-1}

TABLE 3 – GEOLOGICAL MODEL (CASE STUDY)

Layer	Horizontal Permeability	Vertical Permeability	Porosity	Thickness (m)	Initial Sw	Initial So
1	1500	450	0.2	54	0.15	0.85

TABLE 4 – OVERBURDEN AND UNDERBURDEN CHARACTERISTICS (CASE STUDY)

	Thickness (m)	Temperature (°C)	Heat Capacity ($kJ/m^3/K$)	Thermal Conductivity (kJ/D-m-K)
Overburden	60	15	2390	146.88
Underburden	60	15	2390	233.28

TABLE 5 – RELATIVE PERMEABILITY DATA (CASE STUDY)					
Water – Oil			Liquid - Gas		
S_W	K_{RW}	K_{ROW}	S_L	K_{RG}	K_{ROG}
0.15	0.000	1.000	0.15	0.850	0.000
0.20	0.000	0.882	0.20	0.750	0.000
0.25	0.002	0.800	0.25	0.680	0.003
0.30	0.006	0.720	0.30	0.612	0.008
0.35	0.013	0.600	0.35	0.510	0.020
0.40	0.025	0.470	0.40	0.400	0.038
0.45	0.044	0.350	0.45	0.298	0.056
0.50	0.070	0.240	0.50	0.204	0.056
0.55	0.104	0.165	0.55	0.140	0.069
0.60	0.148	0.093	0.60	0.119	0.075
0.65	0.204	0.000	0.65	0.096	0.090
0.70	0.271	0.000	0.70	0.057	0.137
0.75	0.352	0.000	0.75	0.052	0.199
0.80	0.447	0.000	0.80	0.038	0.257
0.85	0.559	0.000	0.85	0.019	0.311
0.90	0.687	0.000	0.90	0.010	0.454
0.95	0.834	0.000	0.95	0.005	0.628
1.00	1.000	0.000	1.00	0.000	1.000

TABLE 6 – CHARACTERIZATION OF OBJECTIVE FUNCTION VALUES WITHIN THE INITIAL SAMPLE (CASE STUDY)				
	MIN ($1.0E3\ m^3$)	**MAX ($1.0E3\ m^3$)**	**Mean ($1.0E3\ m^3$)**	**Standard Deviation ($1.0E3\ m^3$)**
COP	2.23	154.80	89.30	42.07
CSI	4.99	710.55	392.62	200.83
f	0	-0.5418	-0.2907	0.1427

TABLE 7 – ADDITIONAL SAMPLED POINTS (CASE STUDY)							
RUN	**Vertical spacing (m)**	**Pressure (kPa)**	**Enthalpy (kJ/Kg)**	**Subcooling (°C)**	**COP ($1.0E3\ m^3$)**	**CSI ($1.0E3\ m^3$)**	***f***
NEGO1	10	3759.2593	2480.0000	28.0967	154.1330	641.9000	-0.5211
NEGO2	11	3759.2593	2577.9424	28.1310	156.9660	589.0300	-0.5537
NEGO3	11	3746.9136	2578.7654	25.3189	155.5530	603.1300	-0.5418
NEGO4	11	3759.2593	2576.2963	27.6852	141.6320	460.3600	-0.5239
NEGO5	15	3730.4527	2561.2071	28.9769	117.3830	436.5300	-0.4132
NEGO6	7	2425.9259	2265.1852	16.6770	110.6570	434.1600	-0.3809
NEGO7	12	3722.2222	2487.4074	24.9074	153.4730	620.4700	-0.5254
NEGO8	7	3166.6667	2450.0960	13.3276	129.8720	494.7100	-0.4539
NEGO9	10	3168.8005	2450.6752	5.0019	143.5830	615.9600	-0.4784
NEGO10	4	3167.5812	2450.1875	5.0019	95.0670	451.9400	-0.2980
NEGO11	15	3165.2949	2462.3503	5.0019	89.8760	347.1900	-0.3096
NEGO12	10	3165.2949	2451.6507	5.0019	146.5280	611.9200	-0.4943
NEGO13	10	3166.6667	2450.4618	5.0019	121.3610	442.6300	-0.4306
NEGO14	12	3167.7336	2455.7252	7.7776	123.1230	419.4200	-0.4475
NEGO15	14	3161.1797	2456.5280	5.0006	112.1580	452.0100	-0.3820

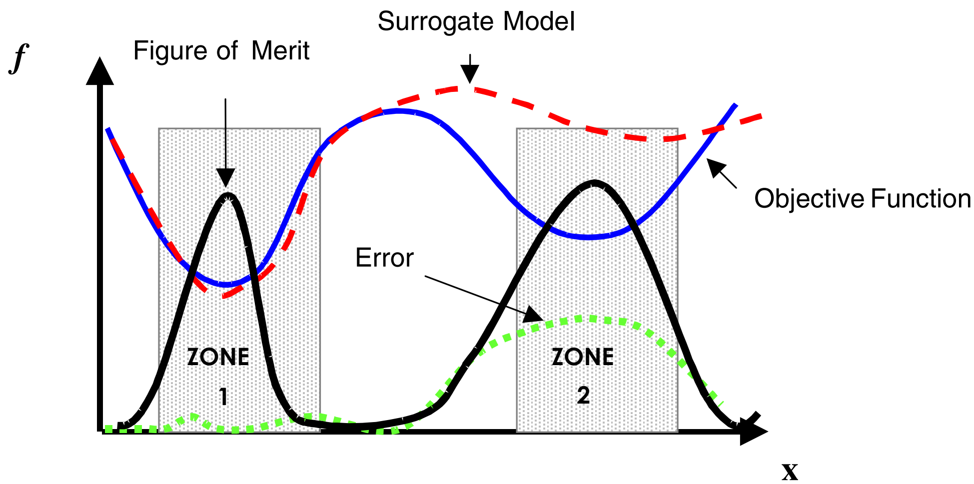

Fig. 1 - Illustration of the purpose of the figure of merit.

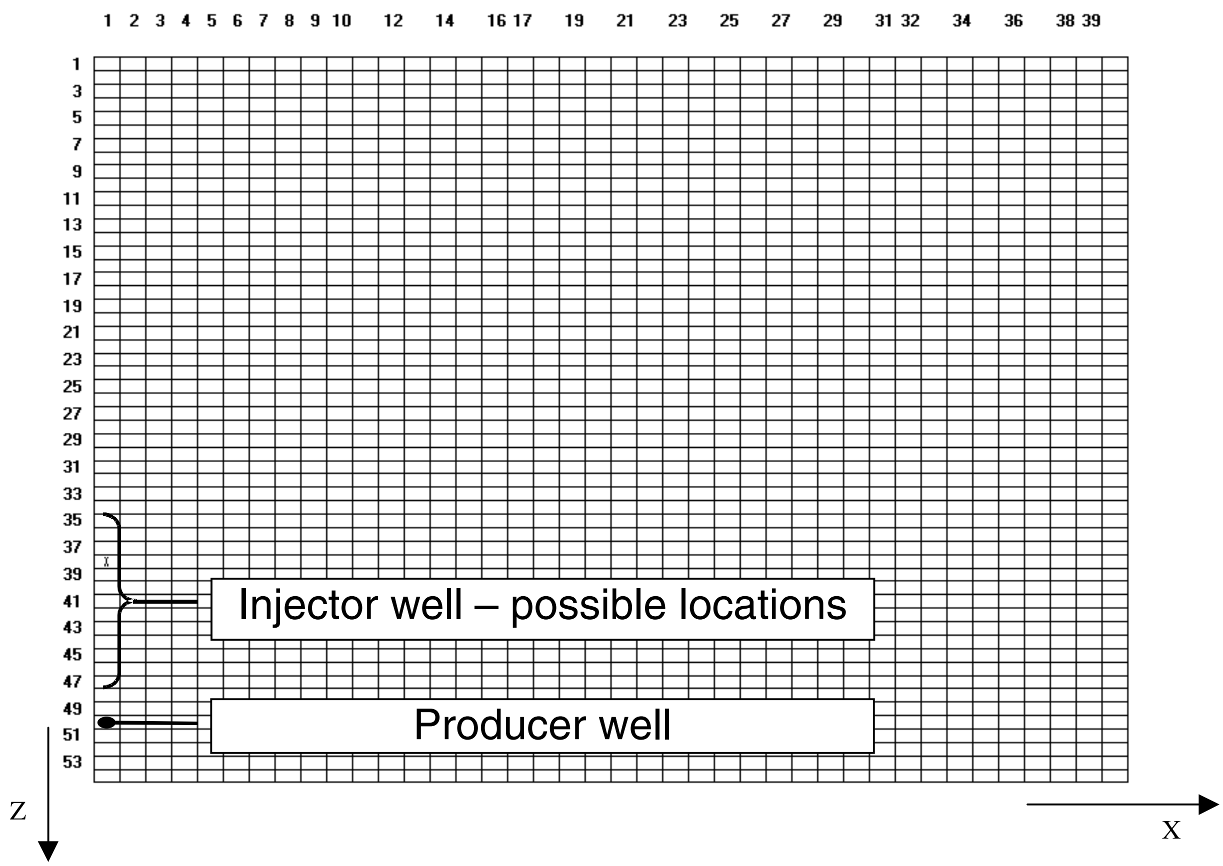

Fig. 2 - Illustration of the grid used in the numerical simulations (Case study).

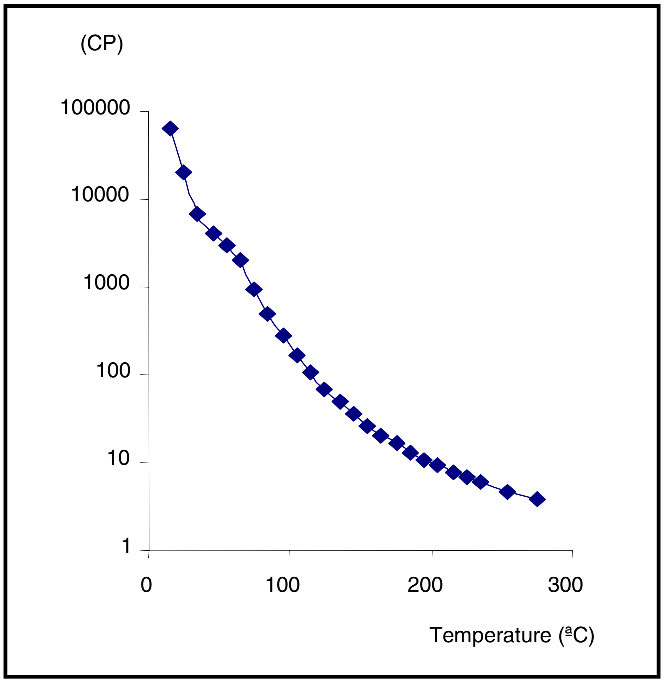

Fig. 3 – Oil viscosity vs. Temperature.