Engineering with Computers (2003) 19: 130–141
DOI 10.1007/s00366-003-0255-1

ORIGINAL ARTICLE

N. V. Queipo · L. E. Zerpa · J. V. Goicochea · A. J. Verde · S. A. Pintos · A. Zambrano

A model for the integrated optimization of oil production systems

Received: 29 August 2001 / Accepted: 19 December

Abstract Typically, the optimization of oil production systems is conducted as a non-systematic effort in the form of trial and error processes for determining the combination of variables that leads to an optimal behavior of the system under consideration. An optimal or near optimal selection of oil production system parameters could significantly decrease costs and add value. This paper presents a solution methodology for the optimization of integrated oil production systems at the design and operational levels, involving the coupled execution of simulation models and optimization algorithms (SQP and DIRECT). The optimization refers to the maximization of performance measures such as revenue present value or cumulative oil production as objective functions, and tubing diameter, choke diameter, pipeline diameter, and oil flow rate as optimization variables. The reference configuration of the oil production system includes models for the reservoir, tubing, choke, separator, and business economics. The optimization algorithms *Sequential Quadratic Programming (SQP)* and *DIRECT* are considered as state-of-the-art in non-linear programming and global optimization methods, respectively. The proposed solution methodology effectively and efficiently optimizes integrated oil production systems within the context of synthetic case studies, and holds promise to be useful in more general scenarios in the oil industry.

Keywords Integrated surface and reservoir modeling and optimization · Oil production systems

Nestor V. Queipo (✉), Luis E. Zerpa, Javier V. Goicochea, Alexander J. Verde, Salvador A. Pintos
Applied Computing Institute, Faculty of Engineering, University of Zulia, Venezuela

Alexander Zambrano
Production Process Optimization, Av. Ernesto Blohm. Edif. PDVSA Exploración y Producción, Chuao. P.O. Box 829. Caracas 1010-A, Venezuela.

Nomenclature

B: Formation volume factor
C_D : Discharge coefficient
D: Non-Darcy skin
D_{CH} :Choke diameter
G: Mass flux
gc : Gravitational constant
h : Height of reservoir
K : Absolute permeability
k : Specific heat ratio, *Cp/Cv*
k_{RO}: Relative permeability of oil
M : Molecular weight
n : Politropic exponent for gas
P : Absolute pressure
P_R: Average reservoir pressure
P_r : Reduced pressure
q: Production rate, flow rate
R : Universal gas constant
r_e: Radius of drainage
R_S: Gas-Oil ratio
r_w: Radius of wellbore
s: Skin factor
S: Saturation
T : Absolute temperature
T_r : Reduced temperature
v : Mole specific volume
V: Reservoir volume
V_G : Specific gas volume
V_L: Specific liquid volume
x : Free gas quality
X : Mole fraction liquid phase
Y : Mole fraction gas phase, pressure ratio

Greek letters

μ: Viscosity
ρ : Density
ϕ : Porosity
ω : acentric factor

Subscripts

1: Inlet
2: Outlet
C : Critical
G : Gas
i : Hydrocarbon component
L : Liquid
m : Mixture
O : Oil
S : Subcritical

Introduction

There is increasing interest in the oil industry in decreasing the cost of and adding value to production processes, due to the decline of conventional oil reserves worldwide and the correspondingly high production costs. Optimization studies in the design and operation of production facilities are critical to address these issues.

Typically, the optimization of oil production systems is conducted as a non-systematic effort in the form of a trial and error process for determining a combination of variables that leads to an optimal behavior of the system under consideration. The development of a formal methodology to achieve the latter is a fundamental step for the optimal or near optimal selection of production system parameters. However, there have been limited efforts in this direction.

Rosenwald and Green [1] presented an optimization procedure for determining the optimum locations of wells in a reservoir, and to determine the proper sequencing of flow rates from those wells that minimized the difference between the production-demand curve and the flow curve actually attained. The method uses a branch-and-bound mixed-integer program in conjunction with a mathematical reservoir model.

Huppler [2] developed a procedure for finding the most profitable gas field production policy to meet a gas sales contract, applying nonlinear programming to the overall problem of production rate scheduling. Also, Huppler used dynamic programming to find the best investment schedule in each reservoir.

See and Horne [3] conducted an optimization study of field operations under a given set of technical and economic constraints. The proposed methodology consisted of a modeling phase that provides an approximately and locally linear model of the reservoir, and an optimization phase that uses a linear programming algorithm to optimize the production schedule of a reservoir for which the producer or injector well locations have already been fixed.

Lang and Horne [4] developed a procedure for the maximization of oil production, using as the decision variables the injection rates and downhole flowing pressure. The procedure involves two steps: (1) the development of a surrogate model of the reservoir performance; and (2) optimization using linear and dynamic programming algorithms.

Carrol [5] conducted an optimization study of an oil production system (drainage area, well, choke, surface facilities), where the principal objective was to demonstrate the effectiveness of non-linear multivariate optimization techniques (modified version of the Newton method and a direct search method called Polytope), on the maximization of the economic benefits of a well, considering as decision variables, separator pressure and tubing diameter.

Fujii and Horne [6] conducted an optimization study of well networks, where multiple production parameters were optimized simultaneously in terms of a profit-based objective function, such as total oil production, net income from the oil production, or oil production present value. The techniques applied in this work were Newton-type methods, the polytope method, and a version of a genetic algorithm. The optimization technique can be used in the design stage of newly developed fields or in the planning of workovers in existing fields.

Welte and Jager [7] presented IPSE (the Integrated Production Simulation Environment), a computer software development that creates a production system modeling environment where the user can quickly create a model of a complete production system for analysis and optimization purposes.

Handley-Schachler et al. [8] introduced a simulation and optimization method for hydrocarbon production networks based on Sequential Linear Programming (SLP). It has been successfully applied to a wide variety of operational problems, including de-bottlenecking, optimization of compressor strategies and determining the optimal lift-gas allocation to networks of gas lifted wells.

This paper presents a solution methodology for the optimization of integrated oil production systems at the design and operational levels, involving the coupled execution of simulation models and optimization algorithms (SQP and DIRECT). The optimization refers to the maximization of performance measures such as revenue present value or cumulative oil production as objective functions, and tubing diameter, choke diameter, pipeline diameter, and oil flow rate as optimization variables. The reference configuration of the oil production system includes models for the reservoir, tubing, choke, separator, and business economics. The optimization algorithms *Sequential Quadratic Programming* (SQP) [9] and *DIRECT* [10] are considered to be the state-of-the-art in non-linear programming and global optimization methods, respectively.

Problem definition

The problem is one of optimizing an integrated oil production system, where two levels of optimization exist: design and operation. With reference to Fig. 1, it can be written

find $\boldsymbol{x} \in X \subseteq R^p$
such that
$f(\boldsymbol{x})$ is maximized

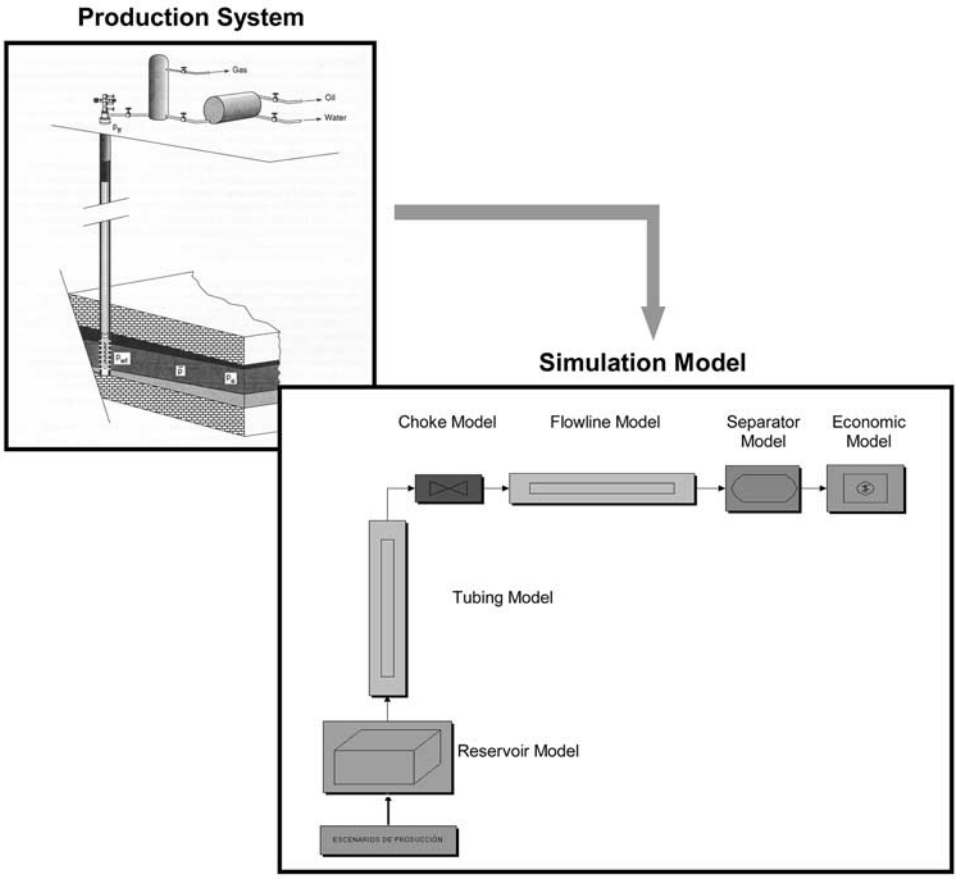

Fig. 1 Oil production system and reference configuration

where f is a mathematical function (objective function) of $\boldsymbol{x}$, the design and/or operation variables vector (tubing diameter, choke diameter, pipeline diameter, separator pressure, and oil production rate), and X is the constraints set. Hence, the problem of interest is one of finding the vector of decision variables that maximize a given performance measure of an oil production system, subject to a set of constraints.

Solution methodology

The proposed solution procedure involves the following steps:

1. Development of a simulation model of an integrated oil production system.
2. Model calibration.
3. Optimization of the production process.

The *simulation model* is a mathematical representation of an integrated oil production system – drainage area, well, wellhead assembly, and surface facilities. Specifically, the model involves several submodels, such as reservoir, multiphase flow in pipes, choke, separator, and business economics (see Fig. 1). The model allows us to evaluate different strategies of reservoir exploitation in terms of cumulative oil production and revenue present value for a given production horizon. A brief explanation of each of the sub-models is given below:

The *Reservoir Model* is a mathematical representation of the dynamic behavior of fluids in the reservoir (porous medium) in terms of pressure and saturation changes. The model consists of a zero-dimensional black oil simulator, based on the mass conservation equation for different phases. The mass conservation equation for each phase relates, for a given time interval, the cumulative mass rate in a volume, with the difference between the inflow and outflow mass rates.

Additionally, the model calculates the flowing bottomhole pressure (P_{wf}) for different oil production rates, using the expression

$$\int_{Pwf}^{P_R} \left(\frac{k_{RO}}{\mu_O B_O}\right) dP = q_O \frac{[ln(r_e/r_w) - 0.75 + s + Dq_O]}{2\pi Kh} \tag{1}$$

The reservoir behavior is simulated by the following solution procedure:

A. Establish initial reservoir pressure, initial phase saturations, oil production rate, and time step length.
B. Calculate the reservoir initial gas and oil mass:

$$\text{Oil mass} \qquad \left(\frac{\phi S_O \rho_O{}^S}{B_O}\right) V \tag{2}$$

$$\text{Gas mass} \qquad \left(\frac{\phi S_G \rho_G{}^S}{B_G} + \frac{\phi S_O R_S \rho_G{}^S}{B_O}\right) V \tag{3}$$

C. Determine the total oil production for the time step (Δt):

$$\Delta N_P = q_O * \Delta t$$

D. Estimate the reservoir pressure at the end of the time step.

E. Calculate the hydrocarbon properties (B_O, B_G, R_S, μ_O, μ_G) for the gas and liquid phases at the new reservoir pressure.

F. Calculate the oil saturation (at time level k):

$$V\left[\left(\frac{\phi S_O \rho_O{}^S}{B_O}\right)_{k-1} - \left(\frac{\phi S_O \rho_O{}^S}{B_O}\right)_k\right] = \Delta N_P \tag{4}$$

G. Calculate the gas saturation:

$$S_G = 1.0 - (S_O + S_W) \tag{5}$$

H. Calculate the oil and gas relative permeabilities as a function of gas saturation:

$$k_{RO} = f(S_G)$$

$$k_{RG} = f(S_G)$$

I. Update the value for oil and gas mass in the reservoir using the saturations calculated in steps F and G.

J. Determine the total gas production for the time step from the total oil production, and the average between the phase mobility ratio and solution gas-oil ratio at times $k - 1$ and k:

$$\Delta G_P = \Delta N_P * \frac{1}{2}\left[\rho_G{}^S \left(\frac{k_{RG}\mu_O B_O}{k_{RO}\mu_G B_G} + R_S\right)_{k-1} + \rho_G{}^S \left(\frac{k_{RG}\mu_O B_O}{k_{RO}\mu_G B_G} + R_S\right)_k\right] \tag{6}$$

K. Calculate gas material balance error, and return to step D, until the error is less than a specified tolerance:

$$Error = \Delta(MasadeGas)_k^{k-1} + \Delta G_P \tag{7}$$

The hydrocarbon properties are determined as a function of API gravity and reservoir pressure from empirical correlations [11–15].

The reservoir model offers:

- Gas production rate.
- Decline of the reservoir pressure with time.
- Change of saturation of the fluids in the reservoir with time.
- Cumulative oil and gas production.
- Inflow Performance Relationship Curve (IPR curve).

The reservoir model consider the following assumptions:

- The reservoir is homogeneous, isotropic, horizontal and of uniform thickness.
- Reservoir model is zero-dimensional.
- There is no flux in the boundaries.
- Cylindrical well model.
- The reservoir drive mechanism is solution gas drive.
- The gas-oil ratio is constant throughout the reservoir.
- Production occurs under pseudo-steady state conditions.
- Effects, such as, capillary pressure, gravity or coning are neglected.

The *Multiphase Flow Model* is a mathematical representation of the fluid transport in the tubing and the horizontal pipeline. Specifically, it calculates the pressure drops in the system due to the flow of multiphase fluids from the bottomhole to the separator for different oil and gas rates and pipe inclinations (horizontal pipeline $\theta = 0°$, vertical tubing $\theta = 90°$), using reported experimental correlations [16]. Further, the model establishes the flowing bottomhole pressure needed to lift the fluids at different production rates for a given separator pressure.

The following algorithm calculates the pressure drop along the hydrocarbon transport system from the well to the separator:

Establish:

- Oil and gas production rates at standard conditions
- Tubing diameter and length
- Horizontal pipeline diameter and length
- Choke diameter
- Flowing bottomhole pressure

A. Calculate wellhead pressure, substracting from the bottomhole pressure the pressure drops along the tubing, and considering that fluid properties are a function of pressure.

B. Calculate the pressure at the choke outlet, from the inlet pressure (wellhead pressure) and the oil and gas production rates.

C. Calculate the inlet separator pressure, from the calculation of pressure drops along the horizontal pipeline from the choke outlet, considering that fluid properties change due to pressure drops.

The procedure for the calculation of pressure drops in pipes is as follows:

a) Divide the pipe in n fragments,

b) For the first fragment, determine the outlet pressure of the element:

b.1) Estimate the outlet pressure,

b.2) Calculate the fluid properties using the average pressure between the inlet and outlet,

b.3) Update the oil and gas flow rates using the average pressure,

b.4) Determine the flow pattern,

b.5) Calculate the liquid fraction,

b.6) Calculate the pressure gradient and obtain the outlet pressure of the element,

b.7) Repeat the steps b.1 to b.6 until the calculated pressure in b.6 is equal to the pressure estimated in b.1.

c) Repeat step (b) for the $n - 1$ remaining fragments of pipe.

The multiphase flow in pipes model estimates the following along the pipeline system:

- Pressure along.
- Flow pattern.
- Oil production rate.
- Gas production rate.

The *Choke Model* is a mathematical representation, originally developed by Sachdeva et al. [17], which models the wellhead choke as a pipe restriction. This model is capable of modeling critical and subcritical flow. In critical flow, the flow rate through the choke reaches a maximum value with respect to the upstream conditions and the fluids equal or exceed the speed of sound. This implies that the flow is choked and the downstream perturbations are unable to propagate upstream; the opposite is also true. For subcritical flow, the flow velocity is less than the speed of sound and the flow rate depends upon the pressure drop through the device, and changes in the upstream pressure affect the downstream pressure.

The choke behavior is simulated by the following solution procedure:

A. Calculate the critical pressure ratio ($Y_C = P_2/P_1$). This is done by iterating and converging on Y_C in the following equation:

$$Y_C = \left\{ \frac{\frac{k}{k-1} + \frac{(1-x)\cdot V_L \cdot (1-Y_C)}{x \cdot V_{G1}}}{\frac{k}{k-1} + \frac{n}{2} + \frac{n\cdot(1-x)\cdot V_L}{x\cdot V_{G2}} + \frac{n}{2}\cdot\left[\frac{(1-x)\cdot V_L}{x\cdot V_{G2}}\right]^2} \right\}^{\frac{k}{k-1}} \tag{8}$$

where

$$V_{G2} = V_{G1} \cdot Y_C^{-\frac{1}{k}} \tag{9}$$

B. Determine the critical mass flux using the critical pressure ratio:

$$G_C = C_D \cdot \left[2 \cdot gc \cdot 144 \cdot P_1 \cdot \rho_{m2}^2 \cdot \left[\frac{(1-x)\cdot(1-Y_C)}{\rho_L} + \frac{x\cdot k}{k-1}\cdot(V_{G1} - Y_C \cdot V_{G2}) \right] \right]^{0.5} \tag{10}$$

$$\rho_{m2} = [x \cdot V_{G1} \cdot Y_C^{-\frac{1}{k}} + (1-x)\cdot V_L]^{-1} \tag{11}$$

C. Determine the mass flux at the choke inlet:

$$G_1 = \frac{\rho_G \cdot q_G + \rho_L \cdot q_L}{\frac{\pi \cdot D_{CH}^2}{4}} \tag{12}$$

D. Compare the mass fluxes from B and C; if the inlet mass flux is greater than the critical mass flux, $G_1 \geq G_C$, then we are in the critical flow region and the maximum outlet pressure is:

$$P_2 = Y_C \cdot P_1 \tag{13}$$

If the calculated mass flux is less than the critical mass flux, $G_1 < G_C$, then we are in the subcritical flow region. The choke outlet pressure is obtained by finding the root (the value of Y_s) of the expression

$$G_1 - G_2 = 0 \tag{14}$$

$$G_2 = C_D \cdot \left[2 \cdot gc \cdot 144 \cdot P_1 \cdot \rho_{m2}^2 \cdot \left[\frac{(1-x)\cdot(1-Y_S)}{\rho_L} + \frac{x\cdot k}{k-1}\cdot(V_{G1} - Y_S \cdot V_{G2}) \right] \right]^{0.5} \tag{15}$$

The expression $G_1 - G_2$ is a parabolic function, as shown in Fig. 2. The proper root is in the range $\{Y_C < Ys < 1.0\}$. When the proper root has been found, the pressure ratio is known and the choke outlet pressure is

$$P_2 = Y_S \cdot P_1 \tag{16}$$

The choke model calculates the pressure drop through the choke. This model considers the following assumptions:

- Flow is one-dimensional.
- Phase velocities are equal at the throat.
- The quality of the mixture is constant for high-speed processes (no mass transfer between the phases).
- The liquid phase is incompressible.

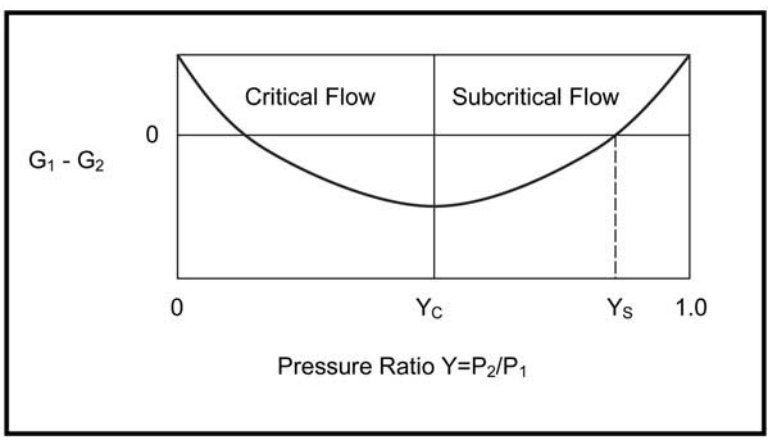

Fig. 2 Proper and false roots of the mass flux differential function – Choke model

The *Separator Model* is a mathematical representation of an isothermal flash separation process or flash equilibria. The input of the model is the liquid and gas composition at the entrance of the separator, and the output is the liquid and gas composition, and the amount of the phases that result of the separation process, at a given temperature and pressure of the separator [18–20].

The separation occurs in a constant pressure process, achieved by controlling the oil and gas flow that exits the separator. The mass transfer between the phases depends on the composition of the hydrocarbon and the operation separator conditions.

In general, the separation process consists of passing the well stream into a separator to isolate the gas and liquid phases. The liquid phase, a result of the separation process, is passed to additional stages of separation that operate at lower pressures until it is deliver to the stock tank.

The liquid fraction recovery is improved by adding more separators between the wellhead and the stock tank. Increasing the number of separators, the separation process is transformed from a flash liberation process to a differential liberation process, maximizing the liquid recovery. For a finite number of separators, there is an optimal combination of separation pressures that maximize the liquid recovery.

In this study, the separation model was implemented for one separation stage, but it could be used to model several separation stages, and to optimize the combination of separator pressures that maximize the liquid fraction recovery.

The procedure of performing a flash calculation is iterative and converges when the fugacity of each component is the same for both phases, as follows:

A. First, determine the number of moles that enter to the separator.

Number of moles of the liquid phase:

$$F_L = \frac{q_{L,1} \cdot \rho_L}{M_L} \tag{17}$$

Number of moles of the gas phase:

$$F_G = \frac{q_{G,1} \cdot \rho_G}{M_G} \tag{18}$$

Total mole number (feed):

$$F = F_L + F_G \tag{19}$$

Been X and Y the liquid and gas composition that enter the separator, respectively.

The overall composition of the mixture that enter the separator is represented by the following expression:

$$Z_i = \frac{F_L \cdot X_i + F_G \cdot Y_i}{F} \tag{20}$$

B. Estimate the gas and liquid phase composition at the separator pressure and temperature conditions, estimating the equilibrium ratio for each component (Wilson equation), and the molar fractions for each phase (Rachford–Rice). The equilibrium ratio (K_i) is the ratio of gas mole fraction (Y_i) to the liquid mole fraction (X_i) of the component I:

$$K_i = \frac{Y_i}{X_i} \tag{21}$$

B.1 The first estimate of the equilibrium ratios is made using the Wilson equation (1962):

$$K_i = \frac{exp\left[5.37(1 + \omega_i)\left(1 - \frac{1}{T_{ri}}\right)\right]}{P_{ri}} \tag{22}$$

B.2 The Rachford–Rice equation expressed as function of gas mole fraction (V) is

$$g(V) = \sum_{i=1}^{n} (Y_i - X_i) = \sum_{i=1}^{n} \frac{Z_i(K_i - 1)}{1 + V(K_i - 1)} = 0 \tag{23}$$

The Rachford–Rice equation expressed as function of liquid mole fraction (L) is

$$f(L) = \sum_{i=1}^{n} \frac{Z_i(1 - K_i)}{L + (1 - L)K_i} = 0 \tag{24}$$

Evaluate the Rachford–Rice equation for gas mole fraction values of $V = 0$ and $V = 1$, to determine the number of phases present. If $g(0) < 0$ (only liquid) or if $g(1) > 0$ (only gas), then for the first iteration the gas mole fraction is fixed to a value of 0.5, in subsequent iterations, if $g(0) < 0$, $V = 0$, and if $g(1) > 0$, then $V = 1$.

If $g(0) > 0$ and $g(1) < 0$, there are two phases (liquid and gas). Using the Newton–Raphson method to solve the Rachford–Rice equation, determine the liquid mole fraction (L) as follows:

$$L^{k+1} = L^k - \frac{f(L^k)}{\frac{\partial f}{\partial L}\Big|L^k} \tag{25}$$

and convergence if achieved when both

1. $abs(L^{k+1} - L^k) < \epsilon$
2. $f(L^{k+1}) < \epsilon$

where ϵ is a small tolerance.

The gas mole fraction is

$$V = 1 - L \tag{26}$$

B.3 Determine the liquid and gas phase compositions, calculating the mole fraction of the components in each phase:

$$X_i = \frac{Z_i}{[1 + V(K_i - 1)]},\ i = 1 \ldots n \tag{27}$$

$$Y_i = \frac{K_i \cdot Z_i}{[1 + V(K_i - 1)]},\ i = 1 \ldots n \tag{28}$$

C. Calculate the equation of state Soave–Redlich–Kwong (SRK) parameters from the critical pressure and temperature of each component (a, b, α).
Equation of state SRK:

$$P = \frac{R \cdot T}{v - b} - \frac{a \cdot \alpha}{v(v + b)} \tag{29}$$

$$b_i = 0.08664 \frac{R \cdot T_{ci}}{P_{ci}} \tag{30}$$

$$a_i = 0.42747 \frac{R^2 \cdot T_{ci}^2}{P_{ci}} \tag{31}$$

$$\alpha_i = [1 + m_i (1 - T_{ri}^{0.5})]^2 \tag{32}$$

$$m_i = 0.48 + 1.574\omega_i - 0.176\omega_i^2 \tag{33}$$

Mixing rules:

$$b_m = \sum_{i=1}^{n} X_i \cdot b_i \tag{34}$$

$$(a\alpha)_{ij} = (1 - k_{ij}) (a\alpha)_i^{0.5} (a\alpha)_j^{0.5} \tag{35}$$

where k_{ij} are the binary interaction parameters, when all the components are hydrocarbons these parameters are zero.

$$(a\alpha)_m = \sum_i \sum_j X_i \cdot X_j (a\alpha)_{ij} \tag{36}$$

If all k_{ij} are zero:

$$(a\alpha)_m = \left(\sum_i X_i (a\alpha)_i^{0.5} \right)^2 \tag{37}$$

The equation of state SRK could be expressed in a cubic form as follows:

$$z^3 - z^2 + (A - B - B^2) z - A \cdot B = 0 \tag{38}$$

where z is the compressibility factor, and A and B are:

$$A = \frac{(a\alpha)_m P}{R^2 \cdot T^2} \tag{39}$$

$$B = \frac{b_m \cdot P}{R \cdot T} \tag{40}$$

D. Solve the equation of state for the compressibility factors of each phase. The major root gives the gas compressibility factor, and the minor positive root gives liquid compressibility factor.

E. Determine the partial fugacities of each component, in each phase. The fugacity coefficient is given by

$$ln\phi_i = \frac{b_i}{b_m} (z - 1) - Ln(z - B) + \frac{A}{B} \left(\frac{b_i}{b_m} - 2 \left(\frac{(a\alpha)_i}{(a\alpha)_m} \right)^{\frac{1}{2}} \right) Ln \left(1 + \frac{B}{z} \right) \tag{41}$$

and the partial fugacities by

$$f_i = X_i \cdot \phi_i \cdot P \tag{42}$$

F. If the partial fugacities ratio of each component is one, then the equilibrium between the two phases is achieved.
The procedure is assume to converge when,

$\sum_i \left(\frac{f_{i,L}}{f_{i,V}} - 1 \right)^2 < \epsilon$, where ϵ is a small tolerance.

If the convergence criterion is not accomplished, estimate new values of the equilibrium ratio as follows:

$$K_i = \frac{\phi_{i,L}}{\phi_{i,V}} \tag{43}$$

then proceed from step B.

G. Calculate the production rates that results from the separation process.
Liquid production rate:

$$q_{L,2} = \frac{L \cdot F \cdot M_L}{\rho_L} \tag{44}$$

gas production rate:

$$q_{g,2} = \frac{V \cdot F \cdot M_G}{\rho_G} \tag{45}$$

In general, the *Economic Models* estimate the profits in a given time period, considering the revenues and costs involved [21]. These models calculate the Net Present Value (NPV) as the present value of the revenue stream (PV_{rev}) minus the present value of the costs (PV_{cost}), where the PV_{rev} reflects the present value of the revenue stream associated with the produced hydrocarbon market value. The PV_{cost} is calculated as the present value of the total cost required to maintain the well production during a given time period. This cost is the sum of operational cost plus tax payments. The operational costs are associated with the operation and maintenance of the well. The cost due to tax payments is estimated based on the present value of the revenue stream, and the valid tax rules. The economic model developed in this study does not consider PV_{cost}.

The present value of the revenue stream is determined by the expression

$$PV_{rev} = \sum_{i=2}^{n} [PV_{rev_{i-1}} + \Delta COP_i \cdot S \cdot D^n] \tag{46}$$

where ΔCOP_i represents the incremental oil cumulative production for the ith production period, n is the number of periods of composition over which the interest rate is applied, S is the oil barrel price, and D is the discount factor. This factor is calculated by the expression

$$D = \frac{1}{1 + \left(\frac{i_a}{n_a} \right)} \tag{47}$$

where i_a is the interest rate and n_a is the number of periods of composition per year.

The cost present value PV_{Cost} is calculated by

$$PV_{Cost} = PV_{COper} + PV_{tax} \tag{48}$$

where PV_{Coper} and PV_{tax} are the operational costs and tax payments, respectively.

Usually, these costs have an annually predefined value. The increase in operational costs is determined based in an annually fixed incremental rate (*IncOp*), by the expression

$$PV_{COper} = COper_1 + \sum_{i=2}^{n} (\mathrm{PV}_{COper} \cdot \mathrm{IncOp} \cdot \mathrm{D}^n) \tag{49}$$

where $COper_1$ is the operational cost in the first year.

The cost present value due to tax payments are estimated by the expression

$$VP_{CImp} = VP_{Ing} \cdot Imp \tag{50}$$

where *Imp* is the impositive rate per oil barrel produced.

Finally, the net present value, *NPV* is determined by

$$NVP = PV_{Ing} - PV_{Cost} \tag{51}$$

Model calibration

The model response (e.g. cumulative production, net present value, etc.) depends upon a series of parameters of the different submodels. These parameters differ in their nature and the uncertainty associated with their specification. For example, the diameter of the well and the average reservoir pressure are parameters of the well and the reservoir model (drainage area), respectively, that typically exhibit different levels of uncertainty. In many cases, it is possible to reduce the uncertainty associated with the specification of these parameters through subsurface and surface tests. Nevertheless, the possibility of conducting all the possible tests is limited by economic considerations. Hence, it is necessary to establish a methodology for the calibration of the model considering production data, or other well known subsurface/surface measures.

The proposed methodology for the simulated model calibration is similar to that known as 'history matching' in reservoir simulation, and consists of formulating the calibration problem as an optimization problem: finding a set of parameters that minimizes the difference between the values calculated by the model and the corresponding historical ones.

Formally, the problem can be written

find $\boldsymbol{x}$ such that,

$$\min \sum_{j=1}^{m} w_j \| f_{obs\,j} - f_{cal\,j}(x) \|^2$$

subject to a set of constrictions of the form

$$x_{i\,min} \leq x_i \leq x_{imax} \qquad i = 1, \ldots, n$$

where

$\boldsymbol{x}$: is the vector of desired model parameter values, for example,

- Porosity (ϕ),
- Relative permeability (k_{ro}, k_{rg}) for a given phase saturation,
- Initial gas saturation (S_{gi}).

f_{obs} and f_{cal}: represent the historical values and the values calculated by the model, for example, gas production rate.

w: denotes a weighting vector for specifying the level of adjustment of the model with respect to the observed measures.

The symbol $\| \cdot \|$ represents the norm used (e.g. euclidean distance) for calculation of the distance between the observed and calculated measures.

The proposed procedure for calibrating the simulation model is:

- Define the set of calibration parameters. These are the parameters that are changed in the model for matching the model output with the corresponding observed field values.
- Define the lower and upper bounds for the calibration parameters.
- Define the initial values of the calibration parameters.
- Define a function, called objective function, which measure the level of matching between the model output and observed field values.

Once the model of the integrated oil production system is calibrated we might proceed with the design and operations optimization process.

Optimization

The proposed solution procedure for the optimization of the integrated oil production system is:

- Define the design and operational parameters, their limits, and initial values.
- Define the objective function. In this study, the objective function is the present value of the revenue stream.
- Define the model parameters that describe the production system.

Figure 3 shows a diagram of the algorithm used for solution of the optimization and calibration problems.

Case study

The proposed methodology is evaluated using a production system with the following characteristics: the reservoir (drainage area) is at a depth of 1212 m (3975 ft.), has an initial pressure of 27579 Kpa (4000 psi), a initial

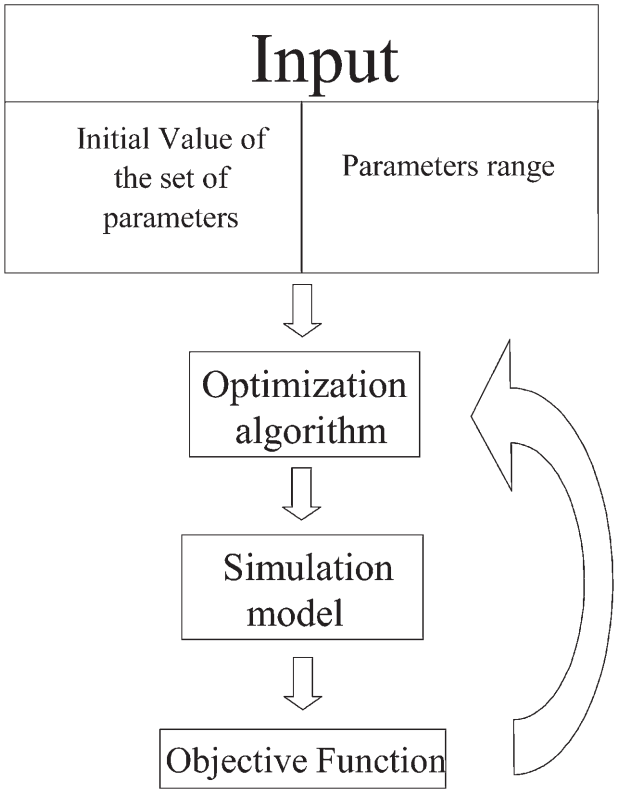

Fig. 3 Diagram of the optimization and calibration procedures

Table 1 Economic parameters, Case Study

Parameter	Value
Interest rate (%)	10
Number of periods of composition per year	12
Oil barrel price ($)	25
Production Horizon (years)	5

saturation of 0.8 and 0.1 for the oil and water (immobile) phases, respectively, with dimensions of 762 × 762 × 15 m, the porosity is assumed constant and equal to 0.3, with a absolute permeability of 100 md, and a hydrocarbon of 20° API. The well is completed at a depth of 1219 m (4000 ft), has an effective diameter of 7.62 cm (3 in), the choke has a diameter of 2.54 cm (1 in), the horizontal pipeline has a length of 304.8 m (1000 ft) and a diameter of 7.62 cm (3 in). The economic and calibration parameters are shown in Tables 1 and 2.

Table 2 Calibration parameters, Case Study

Parameter	Range	Unit
Porosity (ϕ)	0.25–0.40	adim.
Oil relative permeability (k_{ro}) for $S_o = 0.8$	0.50–0.80	adim.
Gas relative permeability (k_{rg}) for $S_g = 0.6$	0.20–0.30	adim.
Gas initial saturation (S_{gi})	0.00–0.10	adim.

Table 3 Decision variables, Case Study

Variables	Range	Unit
Tubing diameter (D_v)	6.35–11.4 (2.5–4.5)	cm (in.)
Pipeline diameter (D_h)	6.35–11.4 (2.5–4.5)	cm (in.)
Choke diameter (D_{ch})	1.27–3.81 (0.5–1.5)	cm (in.)
Oil production rate (q_i)	31.8–636 (200–4000)	m^3/day (bbl/day)
Skin factor (s)	(−3/3)	adim.

At the design level, the decision variables for optimization of the production system are: the vertical tubing diameter, the horizontal pipeline diameter, choke diameter, oil production rates, and skin factor. The objective function is the present value of the revenue stream associated with the market value of the hydrocarbons produced. Table 3 shows the range of the decision variables. Considering as exploitation strategy the production at a constant rate in five periods within the production horizon.

At the operational level, the decision variables for the optimization of the production system are: the oil production rates and the skin factor, using the limits shown in Table 3 of the corresponding decision variables. The objective function is the present value of the revenue stream. Considering the same exploitation strategy explained above.

Results and discussion

The results are evaluated in terms of:

- Optimum values of the objective function obtained by the optimization algorithms.
- Time of optimization process.
- Number of function evaluations.

Results of the calibration problem

In the calibration problem the best value for the objective function was obtained using the SQP method (see Fig. 4).

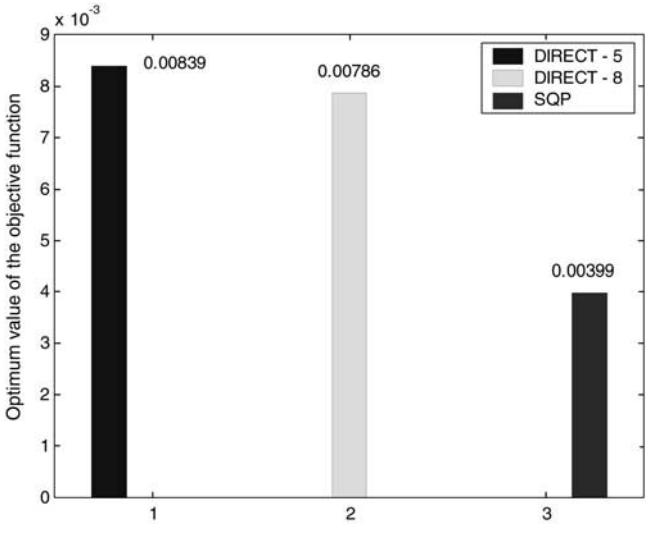

Fig. 4 Optimum value of the objective function obtained by SQP and DIRECT – Case Study

Table 4 Calibration parameter values found by SQP and Direct algorithms - Case Study

DIRECT – 5 iterations			
Initial Point		Best Solution Found	
Porosity	0.325	Porosity	0.25
K_{ro} (S_o = 0.8)	0.65	K_{ro} (S_o = 0.8)	0.78
K_{rg} (S_g = 0.6)	0.25	K_{rg} (S_g = 0.6)	0.25
$S_{g,i}$	0.05	$S_{g,i}$	0.05
Initial Function Value		Objective Function Value	
0.0199		0.0083	
DIRECT – 8 iterations			
Initial Point		Best Solution Found	
Porosity	0.325	Porosity	0.25
K_{ro} (S_o = 0.8)	0.65	K_{ro} (S_o = 0.8)	0.78
K_{rg} (S_g = 0.6)	0.25	K_{rg} (S_g = 0.6)	0.25
$S_{g,i}$	0.05	$S_{g,i}$	0.04
Initial Function Value		Objective Function Value	
0.0199		0.0078	
SQP			
Initial Point		Best Solution Found	
Porosity	0.325	Porosity	0.25
K_{ro} (S_o = 0.8)	0.65	K_{ro} (S_o = 0.8)	0.63
K_{rg} (S_g = 0.6)	0.25	K_{rg} (S_g = 0.6)	0.26
$S_{g,i}$	0.05	$S_{g,i}$	0.0
Initial Function Value		Objective Function Value	
0.0199		0.0039	

Table 5 Best solution found by SQP and Direct algorithms at the design level - Case Study

DIRECT			
Initial Point		Best Solution Found	
D_v (cm)	8.89	D_v (cm)	10.6
D_h (cm)	8.89	D_h (cm)	10.6
D_{ch} (cm)	2.54	D_{ch} (cm)	2.54
q_1 (m^3/day)	334	q_1 (m^3/day)	334
q_2 (m^3/day)	334	q_2 (m^3/day)	132
q_3 (m^3/day)	334	q_3 (m^3/day)	132
q_4 (m^3/day)	334	q_4 (m^3/day)	334
q_5 (m^3/day)	334	q_5 (m^3/day)	334
Skin factor	0	Skin factor	−2
Initial Function Value		Objective Function Value	
4772823.27 $		10844863.13 $	
SQP			
Initial Point		Best Solution Found	
D_v (cm)	8.89	D_v (cm)	9.27
D_h (cm)	8.89	D_h (cm)	8.48
D_{ch} (cm)	2.54	D_{ch} (cm)	2.54
q_1 (m^3/day)	334	q_1 (m^3/day)	341
q_2 (m^3/day)	334	q_2 (m^3/day)	334
q_3 (m^3/day)	334	q_3 (m^3/day)	334
q_4 (m^3/day)	334	q_4 (m^3/day)	334
q_5 (m^3/day)	334	q_5 (m^3/day)	334
Skin factor	0	Skin factor	0
Initial Function Value		Objective Function Value	
4772823.27 $		4931253.93 $	

Table 4 shows the values of the objective function given at the end of the execution of the optimization algorithms and the corresponding parameter values. Further, it shows the values of the initial point and its corresponding objective function value. Note that the objective function measure how good or bad is the adjustment of the calculated values with respect to the observed values, i.e. the lower the objective function, the better the adjustment.

Figure 5 shows the curves of gas production rate versus time calculated by the simulation model using the parameter values that resulted from the calibration process. Note that the best adjustment of the curve of observed values is obtained by the SQP method.

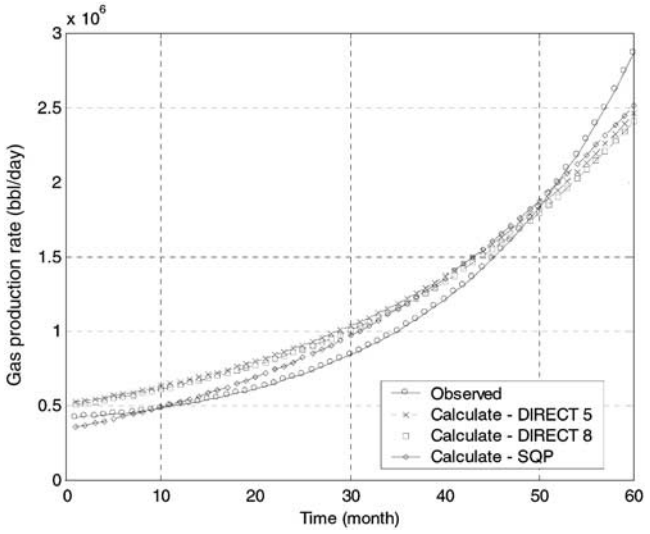

Fig. 5 Gas production rate vs. time for the model calibration results and the field observed values – Case Study

Results of the optimization problem at the design level

The best value of the objective function was obtained using the DIRECT method. Table 5 shows the best values of the objective function found by the optimization methods and the corresponding final parameter values. In addition, it shows the initial point and corresponding objective function value. Table 6 shows the number of iterations, number of function evaluations and execution time of the SQP and DIRECT optimization methods. Note that DIRECT was also the algorithm with lower execution time.

Table 6 Computational cost during optimization process at the design level, Case Study

DIRECT	
Iterations	5
Number of evaluations	127
Process time	41971 sec
SQP	
Iterations	16
Number of evaluations	311
Process time	75145 sec

Table 7 Best solution found by the SQP and Direct algorithms at the operational level - Case Study

DIRECT			
Initial Point		Best Solution Found	
q_1 (m^3/day)	334	q_1 (m^3/day)	334
q_2 (m^3/day)	334	q_2 (m^3/day)	132
q_3 (m^3/day)	334	q_3 (m^3/day)	334
q_4 (m^3/day)	334	q_4 (m^3/day)	334
q_5 (m^3/day)	334	q_5 (m^3/day)	334
Skin factor	0	Skin factor	−2
Initial Function Value		Objective Function Value	
3620641.79 $		8420658.92 $	
SQP			
Initial Point		Best Solution Found	
q_1 (m^3/day)	334	q_1 (m^3/day)	345
q_2 (m^3/day)	334	q_2 (m^3/day)	334
q_3 (m^3/day)	334	q_3 (m^3/day)	334
q_4 (m^3/day)	334	q_4 (m^3/day)	334
q_5 (m^3/day)	334	q_5 (m^3/day)	334
Skin factor	0	Skin factor	0
Initial Function Value		Objective Function Value	
3620641.79 $		3789002.85 $	

The best solution found represents an increase of 127% in the present value of the revenue with respect to the objective function associated with the central point in the decision variable space.

Results of the optimization problem at the operational level

The best value of the objective function was obtained using the DIRECT method. Table 7 shows the best values of the objective function found by the optimization methods and the corresponding final parameter values. In addition, it shows the initial point and corresponding objective function value. Table 8 shows the number of iterations, number of function evaluations, and execution time of the SQP and DIRECT optimization methods. Note that DIRECT was also the algorithm with the lowest execution time.

The best solution found represents an increase of 132% in the present value of the revenue with respect to the objective function associated with the central point in the decision variable space.

Table 8 Computational cost during optimization process at the operational level, Case Study

DIRECT	
Iterations	5
Number of evaluations	147
Process time	38030 sec
SQP	
Iterations	15
Number of evaluations	309
Process time	59131 sec

Conclusions

- This paper presents a solution methodology for the optimization of integrated oil production systems at the design and operational levels, involving the coupled execution of simulation models and optimization algorithms (SQP and DIRECT).
- The *simulation model* is a mathematical representation of an integrated oil production system: drainage area, well, wellhead assembly, and surface facilities. Specifically, the model involves several submodels, such as reservoir, multiphase flow in pipes, choke, separator, and business economics. The model allows us to evaluate different strategies of reservoir exploitation in terms of cumulative oil production and revenue present value for a given production horizon.
- The optimization refers to the maximization of performance measures, such as revenue present value or cumulative oil production as objective functions; and tubing diameter, choke diameter, pipeline diameter, and oil flow rate as optimization variables. The optimization algorithms Sequential Quadratic Programming (SQP) and DIRECT are considered state-of-the-art in non-linear programming and global optimization methods, respectively.
- The proposed solution methodology was evaluated using case studies of optimization in oil production systems at the design and operational levels. The results showed increases in the present values of the revenues of 127% and 132%, with respect to reference values, respectively.
- The proposed solution methodology effectively and efficiently optimized the integrated oil production systems within the context of synthetic case studies, and holds promise to be useful in more general scenarios in the oil industry.

Acknowledgements

The authors gratefully acknowledge financial support provided by the Consejo Nacional de Investigaciones Petroleras (CONIPET) and the Consejo Nacional de Investigaciones Científicas (CONICIT), Venezuela, to conduct this project.

References

1 Rosenwald GW, Green DW (1974) A method for determining the optimum location of wells in a reservoir using mixed-integer programming. Soc Petroleum Eng J February: 44–54
2 Huppler JD (1974) Scheduling gas field production for maximum profit. Soc Petroleum Eng J June: 279–294
3 See BA, Horne RN (1983) Optimal reservoir production scheduling by using reservoir simulation. Soc Petroleum Eng J October: 717–726
4 Lang ZX, Horne RN (1983) Optimum Production Scheduling Using Reservoir Simulators: A Comparison of Linear Programming and Dynamic Programming Techniques. SPE 12159
5 Carroll JA (1990) Multivariate Production System Optimization. MS thesis, Stanford University, Stanford, CA

6 Fujii H, Horne R (1995) Multivariate optimization of networked production systems. SPE Production & Facilities August: 165–171
7 Welte KA, Jager J (1996) IPSE: A Production System Modeling Toolkit for Shell Expro. SPE 35557
8 Handley-Schachler S, McKie C, Quintero N (2000) New Mathematical Techniques for the Optimisation of Oil & Gas Production Systems. SPE 65161
9 Papalambros PY, Wilde DJ (1988) Principles of Optimal Design (Modeling and Computation). Cambridge University Press, pp 365–372
10 Jones D, Perttunen C, Stuckman B (1993) Lipschitzian optimization without the Lipschitz constant. J Optimization Theory Applic 79(1): 157–181
11 Vasquez M, Beggs HD (1977) Correlations for fluid physical property prediction. SPE 6719
12 Beggs HD, Robinson JR (1975) Estimating the viscosity of crude oil systems. JPT Forum. SPE 5434
13 Smith CR, Tracy GW, Farrar RL (1992) Applied Reservoir Engineering, Vol I, 1st ed.. OGCI Publications
14 Lee AL, González MH, Eakin BE (1966) The viscosity of natural gases. SPE 1340
15 Sutton RP (1985) Compressibility Factors for High-Molecular-Weight Reservoir Gases. Paper SPE 14265
16 Beggs HD, Brill JP (1973) A study of two-phase flow in inclined pipes. SPE 4007
17 Sachdeva R, Schmidt Z, Brill JP, Blais RM (1986) Two-phase flow through chokes. SPE 15657
18 Abhvani AS, Beaumont DN (1987) Development of an efficient algorithm for the calculation of two-phase equilibria. Paper SPE 13951
19 Henly EJ, Seader JD (1981) Equilibrium – Stage Separation Operations in Chemical Engineering. Wiley, Chichester, p 270
20 Nghiem LX, Aziz K, Li YK (1983) A Robust Iterative Method for Flash Calculations Using the Soave-Redlich-Kwong or the Peng-Robinson Equation of State. SPE 8285
21 Veatch R (1986) Economics of Fracturing: Some Methods, Examples, and Case Studies. SPE 15509